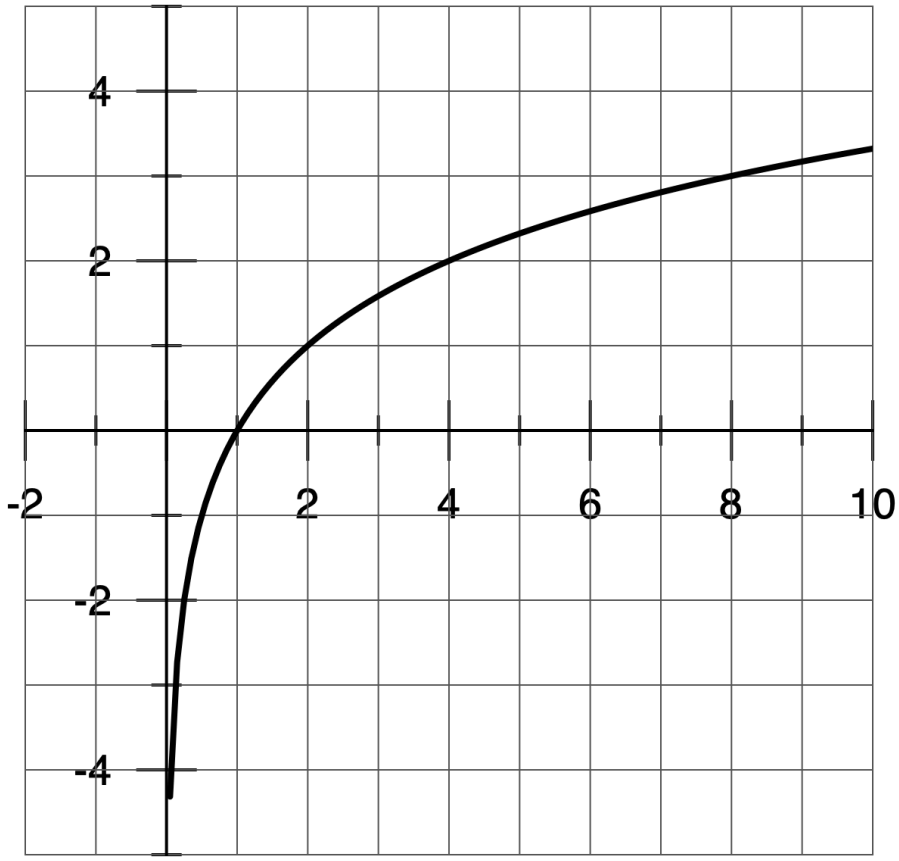
Graphing Logarithmic Functions

These notes include material from section 5.4 (p. 375 – 380) and section 5.6 (p. 401 – 404) in your workbook. You should also read these sections for more complete explanations and additional examples.

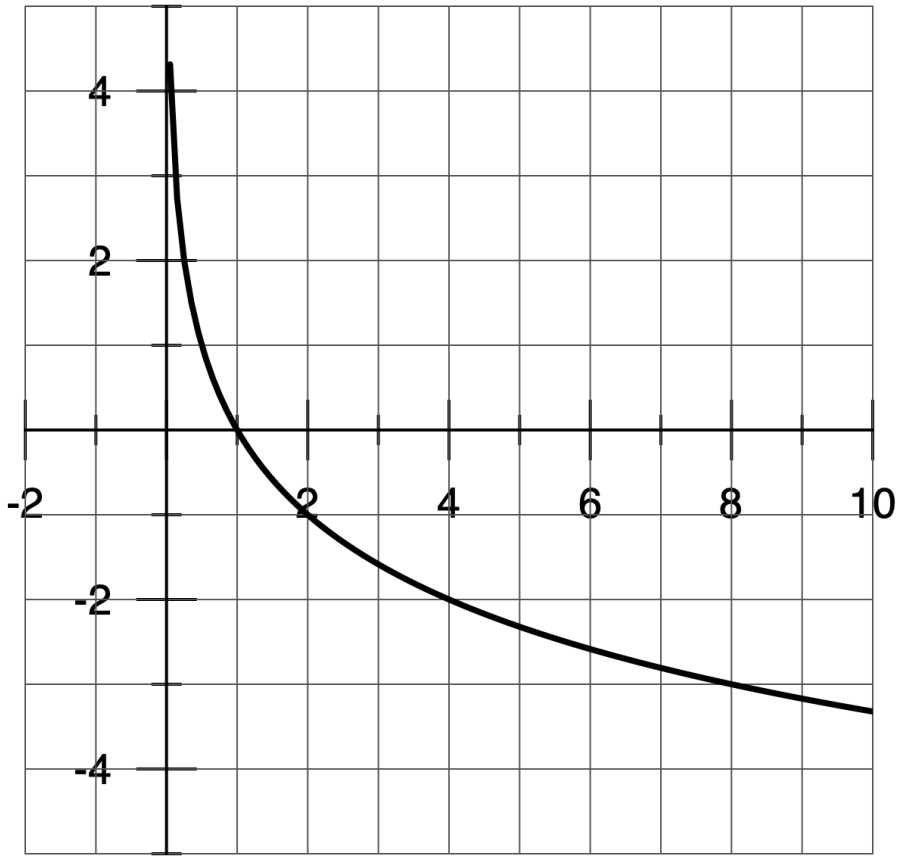
Graphing Logarithmic Functions

In general, the graphs of logarithmic functions have one of two appearances:

1. The graph of $f(x) = \log_a x$ when $a > 1$.



2. The graph of $f(x) = \log_a x$ when $0 < a < 1$.



Note the following properties of the graph of $f(x) = \log_a x$:

1. When $a > 1$, y increases as x increases. The function is said to be **increasing**.
2. When $0 < a < 1$, y decreases as x increases. The function is said to be **decreasing**.
3. The x-intercept of the graph is always 1.
4. The point $(a,\ 1)$ is always on the graph.
5. The y-axis $(x = 0)$ is a vertical asymptote.
6. The graph has no y-intercept.
7. The domain of the function is $x > 0$.
8. The range of the function is $y \in \mathbb{R}$.

To draw the graph of a logarithmic function of the form $f(x) = \log_a x$,

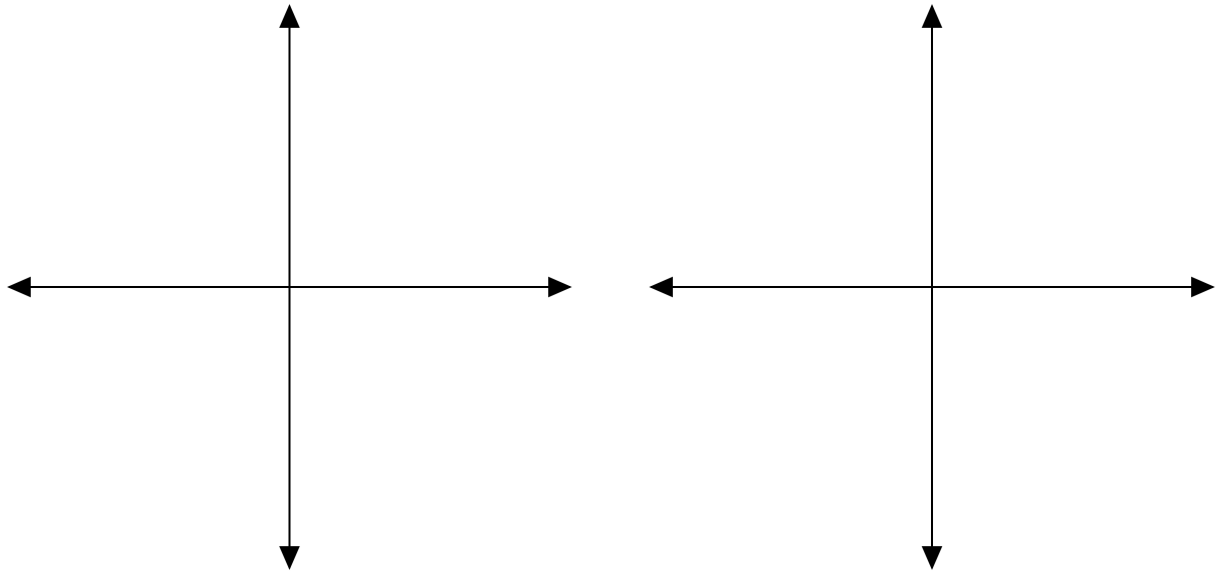
1. Rewrite the logarithmic function as an exponential function.

2. Sketch the graph of the exponential function.

3. Draw the inverse of the exponential function (which is the logarithmic function) by interchanging the x and y coordinates of each key point.

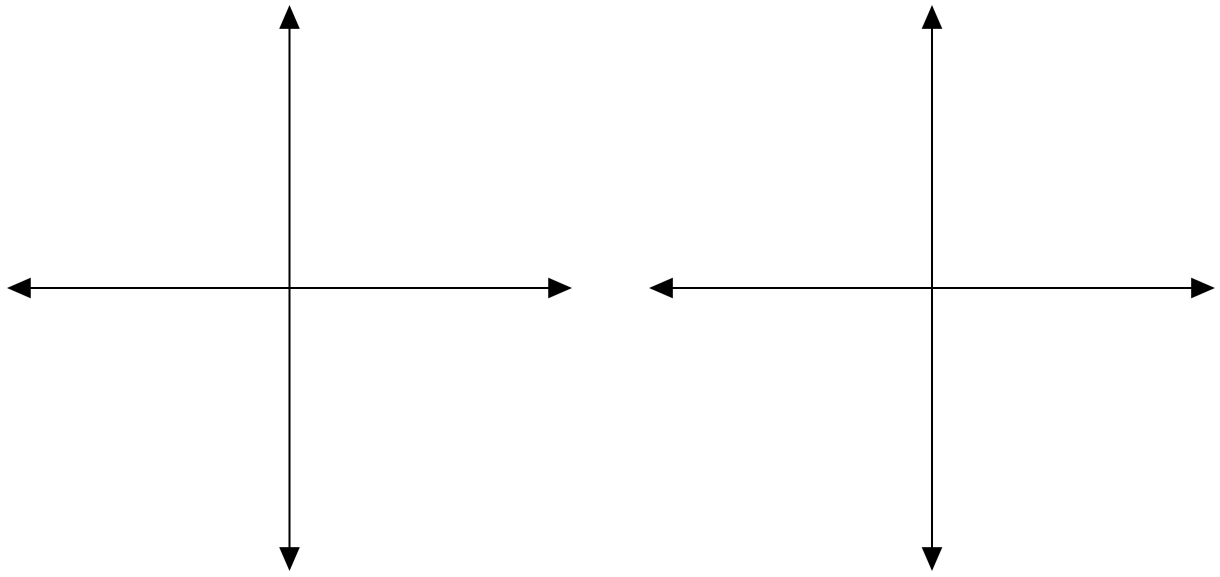
Example (5.4 Ex 4, sidebar p. 379)

a) Graph $y = \log_4 x$.



b) Identify the intercepts, the equations of any asymptotes, and the domain and range of the function.

Example (not in workbook)
Graph $y = \ln x$.



Transforming Logarithmic Functions

The image graph $y = c\log_a b(x-h)+k$ is the graph of $y = a^x$:

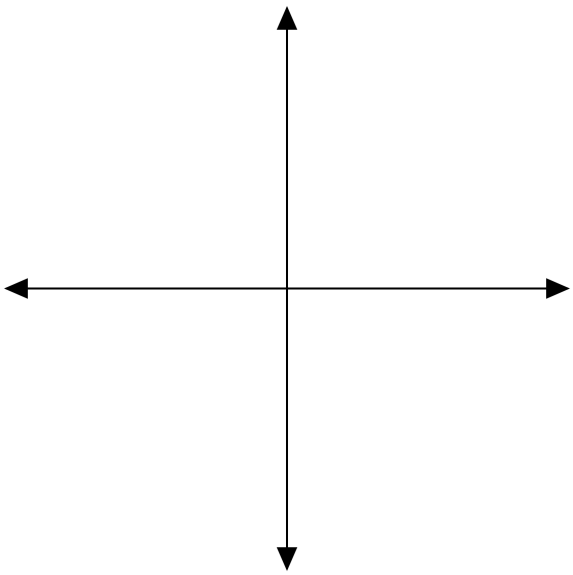
- stretched vertically by a factor of $|c|$
- stretched horizontally by a factor of $\frac{1}{|b|}$
- reflected in the x-axis when $c < 0$
- reflected in the y-axis when $b < 0$
- translated k units vertically
- translated h units horizontally

Note: Remember that transformations must be applied in the correct order (stretches, then reflections, then translations).

Example (5.6 Ex 3, sidebar p. 403)

a) Create a table of values for $y = \log_2 x$.

b) How is the graph of $y = \log_2 x - 1$ related to the graph of $y = \log_2 x$? Sketch these two graphs on the same grid.



c) Identify the intercepts and the equation of the asymptote of the graph of $y = \log_2 x - 1$, and the domain and range of the function.

Example (not in workbook)

Graph $y = 4\log_3(x+1) - 5$.

Example (not in workbook)

Graph $y = 2\ln\left(\frac{1}{2}x + 2\right) - 3$

Homework: #10, 11 in the section 5.4 exercises (p. 381 – 385). Answers on p. 386.
#8 – 11 in the section 5.6 exercises (p. 405 – 410). Answers on p. 411.